# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

## **B.Sc. DEGREE EXAMINATION – STATISTICS**

FOURTH SEMESTER - APRIL 2010

#### ST 4502/ST 4501 - DISTRIBUTION THEORY

Date & Time: 21/04/2010 / 9:00 - 12:00 Dept. No.

Answer ALL Questions

## <u> PART – A</u>

(10 x 2 = 20 marks)

- 1. Show that f(x,y) = 8xy, 0 < x < y < 1 is a joint probability density function (pdf) of a two dimensional R.V. (X,Y).
- 2. For a two dimensional continuous R.V. (X,Y), define the marginal probability density function (pdf) of X and the conditional pdf of Y given X.
- 3. Give the density function of the Hypergeometric distribution.
- 4. Obtain the moment generating function of the geometric distribution and hence find its mean.
- 5. State any two properties of the bivariate normal distribution.
- 6. Find the mean of the Beta distribution of the first kind with parameters  $\alpha$  and  $\beta$ .
- 7. Obtain the pdf of Y = 2X + 1, where X has pdf f(x) = 1/3, x=1,2,3 and 0 elsewhere.
- 8. Write down the t-statistic.
- 9. Find the cumulative distribution function of the largest order statistic of i.i.d. R.V's of the continuous type and hence find its pdf.
- 10. Define the limiting distribution of a random variable.

#### <u> PART – B</u>

Answer any FIVE Questions

- 11. Let f(x,y) = 2, 0 < x < y < 1 be the joint pdf of (X,Y). Obtain the conditional pdf, and hence the conditional mean and variance of X given Y.
- 12. Show that Poisson distribution is a limiting case of the Binomial distribution. Also state and prove the reproductive property of the Poisson distribution.
- 13. Find the mean and variance of the double exponential distribution.
- 14. Obtain the distribution of the random variable  $X^2$ , where  $X \sim N(0,1)$ .
- 15. State and prove the "Memory less property" of the exponential distribution.
- 16. Obtain the marginal distribution of Y if (X,Y) follows a bivariate normal distribution.
- 17. Show that  $X_{(1)}$  follows exponential distribution with parameter  $n\lambda$  if  $X_i$  s are i.i.d. exponential with parameter  $\lambda$ .
- 18. Write a note on (a) multinomial probability distribution (b) stochastic convergence. (P.T.O)

 $(5 \times 8 = 40 \text{ marks})$ 

Max.: 100 Marks

## <u>PART – C</u>

Answer any TWO Questions

- 19. State and prove the classical central limit theorem.
- 20. Obtain the distribution of the sample mean and sample variance from a normal population and show that they are independently distributed.
- 21. Derive the probability density function of Snedecor's F distribution.
- 22. Let f(x,y) = (1 + xy)/4, |x| < 1 & |y| < 1, be the joint pdf of (X,Y). Show that X and Y are not independent but X<sup>2</sup> and Y<sup>2</sup> are independent.

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 $(2 \times 20 = 40 \text{ marks})$